



Quotient projections as equivariant fibrations ☆

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Abstract

We will consider the canonical projection $q : G \rightarrow G/H$ as an H -map with the action by conjugations and prove that it is an H -fibration provided G is an almost connected metrizable group and H is a compact subgroup of G . As a consequence, any G -map $E \rightarrow G/H$ has the equivariant homotopy lifting property with respect to any G -space.

Introduction

Let G be a topological group. By a G -fibration we mean the equivariant version of a Hurewicz fibration, that is to say, a G -map $p : E \rightarrow B$ that has the equivariant homotopy lifting property with respect to any G -space. An important result in the theory of G -spaces is that any G -map $E \rightarrow G/H$ is a G -fibration whenever G is a compact Lie group (see e.g. [5, Proposition 3.1]). This result was generalized in [11, Theorem 4.4] to the actions of non-compact Lie groups and in [4, Corollary 6.5] to compact metrizable groups. At the end of this paper we will prove a non-compact version of this fact for almost connected metrizable groups (Theorem 5.2).

Given a topological group G and its closed subgroup H , we will consider the canonical projection $q : G \rightarrow G/H$, defined as $g \mapsto gH$, as an H -map with respect of the actions of H on G and G/H defined by conjugations. Namely, for every $h \in H$, $g \in G$ and $gH \in G/H$ we put $h_*g = hgh^{-1}$ and $h \cdot gH = hgH$.