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Dynamic properties of the dynamical system $(\mathscr{F}_n^K(X), \mathscr{F}_n^K(f))$

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Abstract

Let (X, f) be a dynamical system, where X is a nondegenerate continuum and f is a map. For any positive integer n, we consider the hyperspace $\mathscr{F}_n(X)$ with the Vietoris topology. For $n > 1$ and $K \in \mathscr{F}_n(X)$ the subset $\mathscr{F}_n(K, X)$ of $\mathscr{F}_n(X)$ is defined as the collection of elements of $\mathscr{F}_n(X)$ containing K. We consider the quotient hyperspace $\mathscr{F}_n^K(X) = \mathscr{F}_n(X) / \mathscr{F}_n(K, X)$, which is obtained from $\mathscr{F}_n(X)$ by shrinking $\mathscr{F}_n(K, X)$ to one point set. Furthermore, we consider the induced maps $\mathscr{F}_n(f) : \mathscr{F}_n(X) \to \mathscr{F}_n(X)$ and $\mathscr{F}_n^K(f) : \mathscr{F}_n^K(X) \to \mathscr{F}_n^K(X)$. In this paper, we introduce the dynamical system $(\mathscr{F}_n^K(X), \mathscr{F}_n^K(f))$ and we study relationships between the conditions $f \in \mathscr{M}$, $\mathscr{F}_n(f) \in \mathscr{M}$ and $\mathscr{F}_n^K(f) \in \mathscr{M}$, where $\mathscr{M}$ is one of the following classes of maps: transitive, mixing, weakly mixing, totally transitive, exact, exact in the sense of Akin-Auslander-Nagar, strongly transitive in the sense of Akin-Auslander-Nagar, exact transitive, fully exact, strongly exact transitive, strongly product transitive, orbit-transitive, Devaney chaotic, irreducible, TT_{++}, strongly transitive and very strongly transitive.

Introduction

Given a continuum X and a positive integer n, the hyperspace $\mathscr{F}_n(X)$, denominated n-fold symmetric product of X, and endowed with the Vietoris topology, consists of all nonempty subsets of X with at most n points [1]. It is known that $\mathscr{F}_n(X)$ is a continuum [2, Corollary 1.8.8]. If