

PID controller tuning for unstable processes using a multi-objective optimisation design procedure

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Abstract: Multi-objective optimisation techniques have shown to be a useful tool for controller tuning applications. Such techniques are useful when: 1) it is difficult to find a controller with a desirable trade-off between conflictive objectives; or 2) it is valuable to extract an additional knowledge from the process by analysing trade-off among possible controllers. In this work, we propose a multi-objective optimisation design procedure for unstable process, using PID controllers. The provided examples show the usability of the procedure for this kind of process, sometimes difficult to control; comparison with existing tuning rule methods provide promising results for this tuning procedure.

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Keywords: PID controller tuning, multi-objective optimisation, evolutionary algorithms.

1. INTRODUCTION

Proportional - Integral - Derivative (PID) controllers remain as reliable and practical control solutions for several industrial processes (Åström and Hägglund, 2001). One of the main advantages of PID controllers is their ease of implementation as well as their tuning, giving a good trade-off between simplicity and cost to implement (Stewart and Samad, 2011). Owing to this, research for new tuning techniques is an ongoing research topic (Åström and Hägglund, 2005). Current research points to guarantee reasonable stability margins as well as a good overall performance for a wide variety of processes.

New tuning techniques are being focused on the fulfilment of several objectives and requirements, sometimes in conflict among them (Ang et al., 2005; Li et al., 2006). Some tuning procedures are based on optimisation statements (Ge et al., 2002; Toscano, 2005; Åström et al., 1998; Panagopoulos et al., 2002; Rajinikanth and Latha, 2012) and some cases they are solved by means of evolutionary algorithms (Reynoso-Meza et al., 2013b). Recently Multi-objective Optimisation (MOO) techniques have shown to be a valuable tool for controller tuning applications (Reynoso-Meza et al., 2014b,a). They enable the designer or decision maker (DM) having a close embedment into the tuning process since it is possible to take into account each design objective individually; they also enable comparing design alternatives (*i.e.* different controllers), in order to select a tuning fulfilling the expected trade-off among conflicting objectives.

As identified in Arrieta et al. (2011), efforts are particularly concentrated in open loop stable systems; nevertheless some critical processes as continuous stirred tank reactors and bioreactors, common in chemical processing units and biological processes, are unstable open loop systems. Several works have been focused on PID-like controllers tuning for such processes; nevertheless, efforts to merge multi-objective optimisation techniques have been not yet applied for such instances.

In this paper, a simple multi-objective problem statement is defined for unstable first order plus dead time (UFOPDT) processes and compared with existing tuning rules. The remainder of this paper is as follows: firstly in Section 2 it is presented a brief background on PID control tuning, UFOPDT process and multi-objective optimisation. Afterwards, a MOO procedure for is presented in Section 3 and it is compared and validated in Section 4. Finally, some concluding remarks are given.

2. BACKGROUND

In order to describe the tuning approach of this paper, some preliminaries in control tuning, unstable open loop process and EMO are required. They are provided below.

2.1 Background on PID controller tuning and unstable process

A basic control loop is depicted in Figure 1. It comprises transfer functions $P(s)$ and $C(s)$ of a process and a controller respectively. The major aim of this control loop

is to keep the desired output $Y(s)$ of the process $P(s)$ in the desired reference $R(s)$.

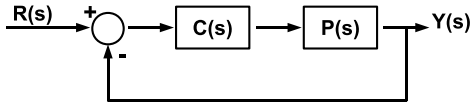


Fig. 1. Basic control loop.

For this work, process $P(s)$ in figure 1 represents a UFOPDT process:

$$P(s) = \frac{K}{Ts - 1} e^{-Ls} \quad (1)$$

where K is the process gain; T the time constant and L the time lag of the system. Equation (2) shows the transfer function of the selected structure of the PID controller:

$$C(s) = k_p \left(1 + \frac{1}{T_I \cdot s} + T_D \cdot s \right) \quad (2)$$

where k_p is the proportional gain, T_I the integral time (s), T_D the derivative time (s); this controller will send a control signal to the process, according with the error $E(s) = R(s) - Y(s)$.

The control problem consists in selecting proportional, integral and derivative gains (k_p , $k_I = \frac{k_p}{T_I}$ and $k_D = k_p \cdot T_D$ respectively) for the PID controller $C(s)$, in order to achieve a desirable performance of the process $P(s)$ in the control loop as well as robust stability margins. Conflicting objectives may appear, when seeking for a desirable trade-off between performance and robustness; for this reason, EMO techniques could be appealing for PID controller tuning.

2.2 Multi-objective optimisation statement

As referred in Miettinen (1998), a multi-objective problem (MOP) with m objectives¹, can be stated as follows:

$$\min_{\theta} \mathbf{J}(\theta) = [J_1(\theta), \dots, J_m(\theta)] \quad (3)$$

subject to:

$$\mathbf{K}(\theta) \leq 0 \quad (4)$$

$$\mathbf{L}(\theta) = 0 \quad (5)$$

$$\underline{\theta}_i \leq \theta_i \leq \bar{\theta}_i, i = [1, \dots, n] \quad (6)$$

where $\theta = [\theta_1, \theta_2, \dots, \theta_n]$ is defined as the decision vector with $\dim(\theta) = n$; $\mathbf{J}(\theta)$ as the objective vector and $\mathbf{K}(\theta)$, $\mathbf{L}(\theta)$ as the inequality and equality constraint vectors respectively; $\underline{\theta}_i, \bar{\theta}_i$ are the lower and the upper bounds in the decision space.

It has been noticed that there is not a single solution in MOPs, because there is not generally a better solution in all the objectives. Therefore, a set of solutions, the Pareto

¹ A maximisation problem can be converted to a minimisation problem. For each of the objectives that have to be maximised, the transformation: $\max J_i(\theta) = -\min(-J_i(\theta))$ could be applied.

set, is defined. Each solution in the Pareto set defines an objective vector in the Pareto front. All the solutions in the Pareto front are a set of Pareto optimal and non-dominated solutions:

- Pareto optimality (Miettinen, 1998): An objective vector $\mathbf{J}(\theta^1)$ is Pareto optimal if there is not another objective vector $\mathbf{J}(\theta^2)$ such that $J_i(\theta^2) \leq J_i(\theta^1)$ for all $i \in [1, 2, \dots, m]$ and $J_j(\theta^2) < J_j(\theta^1)$ for at least one j , $j \in [1, 2, \dots, m]$.
- Dominance (Coello and Lamont, 2004): An objective vector $\mathbf{J}(\theta^1)$ is dominated by another objective vector $\mathbf{J}(\theta^2)$ iff $J_i(\theta^2) \leq J_i(\theta^1)$ for all $i \in [1, 2, \dots, m]$ and $J_j(\theta^2) < J_j(\theta^1)$ for at least one j , $j \in [1, 2, \dots, m]$. This is denoted as $\mathbf{J}(\theta_2) \preceq \mathbf{J}(\theta_1)$.

To successfully implement the multi-objective optimisation approach, three fundamental steps are required: the MOP definition, the multi-objective optimisation (MOO) process and the multi-criteria decision making (MCDM) stage. This integral and holistic process will be denoted hereafter as a multi-objective optimisation design (MOOD) procedure.

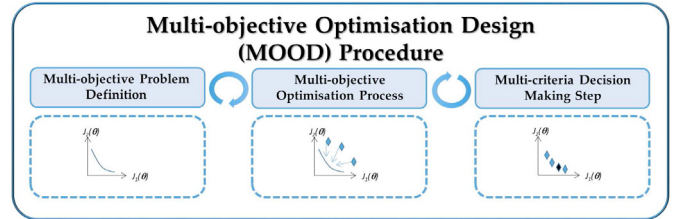


Fig. 2. Multi-objective optimisation design (MOOD) procedure.

Next, this MOOD procedure will be used in order to find suitable PID parameters for UFOPDT processes.

3. MOOD PROCEDURE FOR UNSTABLE SYSTEMS

As commented before, for a successful implementation of the MOOD procedure, the following steps should be carried out: the MOP definition, the optimisation process and the MCDM stage. All of them will be clarified within the PID controller tuning for a UFOPDT framework.

3.1 Multi-objective problem definition

Within this context, the decision variables for the optimisation statement are $\theta = [k_p, T_I, T_D]$. A total of three design objectives will be stated: one related to performance and two related with robustness. In the first case, the settling time $St[s]$ for a step response will be used; in the latter case, the inverse² of gain and phase margins, Gm Pm respectively. It is possible to incorporate more design objectives, nevertheless this would lead to which is known as a many-objectives optimisation instance (Ishibuchi et al., 2008). Such instances represent a particular challenge for MOO algorithms, since convergence and spreading capabilities are usually in conflict. This instance will be addressed in a future work, and we will focus on this paper in a multi-objective problem with 3 design objectives.

² in order to use an overall minimisation problem statement.

$$J_{St}(\boldsymbol{\theta}) = \text{SettlingTime}_{98\%}[s] \quad (7)$$

$$J_{Gm^{-1}}(\boldsymbol{\theta}) = \frac{1}{Gm} [dB^{-1}] \quad (8)$$

$$J_{Pm^{-1}}(\boldsymbol{\theta}) = \frac{1}{Pm} [\text{degrees}^{-1}] \quad (9)$$

Therefore, the following optimisation (minimisation) statement is defined:

$$\min_{\boldsymbol{\theta}} \mathbf{J}(\boldsymbol{\theta}) = [J_{St}(\boldsymbol{\theta}), J_{Gm^{-1}}(\boldsymbol{\theta}), J_{Pm^{-1}}(\boldsymbol{\theta})] \quad (10)$$

subject to:

$$\begin{aligned} k_p &\in [k_p, \overline{k_p}] \\ T_I &\in [T_I, \overline{T_I}] \\ T_D &\in [T_D, \overline{T_D}] \\ J_{St}(\boldsymbol{\theta}) &\leq \overline{St} \\ J_{Gm^{-1}}(\boldsymbol{\theta}) &\leq \overline{Gm^{-1}} \\ J_{Pm^{-1}}(\boldsymbol{\theta}) &\leq \overline{Pm^{-1}} \end{aligned} \quad (11)$$

Last constraints are known as *pertinency* bounds in the objective space, in order to guarantee practical and reasonable controllers in the approximated Pareto front. In order to state such pertinency bounds, an idea on the tolerable value on design objectives is required. In every instance, internal stability of the closed loop is also included as constraint.

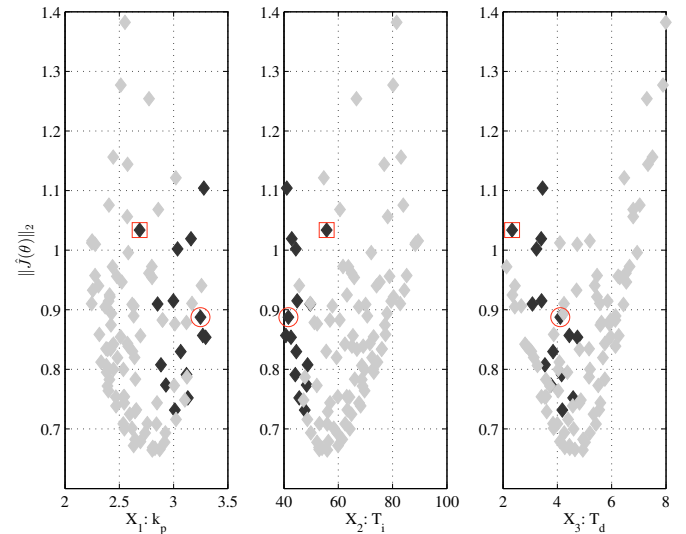
3.2 Multi-objective optimisation process

For the optimisation stage, the sp-MODE algorithm (Reynoso-Meza et al., 2010) will be used³. It is an algorithm based on the differential evolution algorithm (Storn and Price, 1997; Das and Suganthan, 2010) which uses a spherical grid in order to prune the approximated set of solutions, and thus promoting diversity along the Pareto front approximation. It includes also a mechanism to improve pertinency (described in Reynoso-Meza et al. (2012)). Therefore, the algorithm will seek actively for a Pareto front approximation inside the pertinency bounds. Default evolutionary parameters are used in this work.

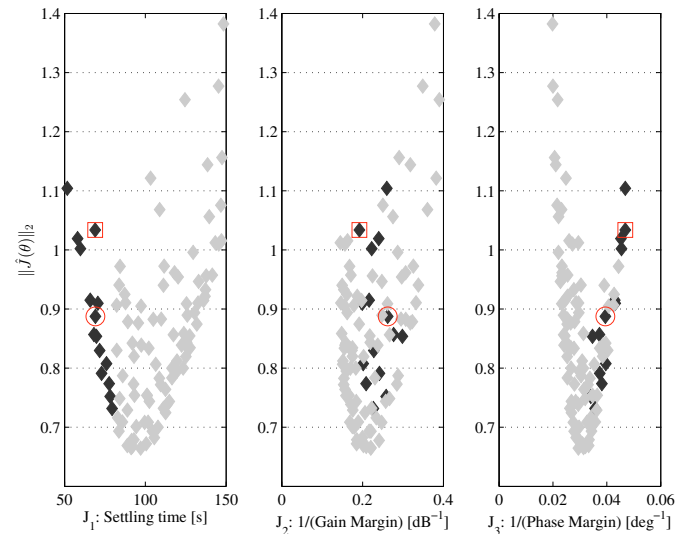
3.3 Multi-criteria decision making stage

In order to visualise and analyse the Pareto front approximation, Level diagrams are used (Blasco et al., 2008; Reynoso-Meza et al., 2013a); they are used due to its capabilities to represent trade-off in the objective and decision space simultaneously. In order to compare specific controllers (up to 5), parallel coordinates are used (Inselberg, 1985) due to its simplicity. Interested readers may refer to Tušar and Filipič (2015) for a review on visualisation tools and techniques.

³ Scripts and tutorial available in Matlab Central at [.../matlabcentral/fileexchange/39215](http://matlabcentral/fileexchange/39215)



(a) Pareto Set Approximation



(b) Pareto Front Approximation

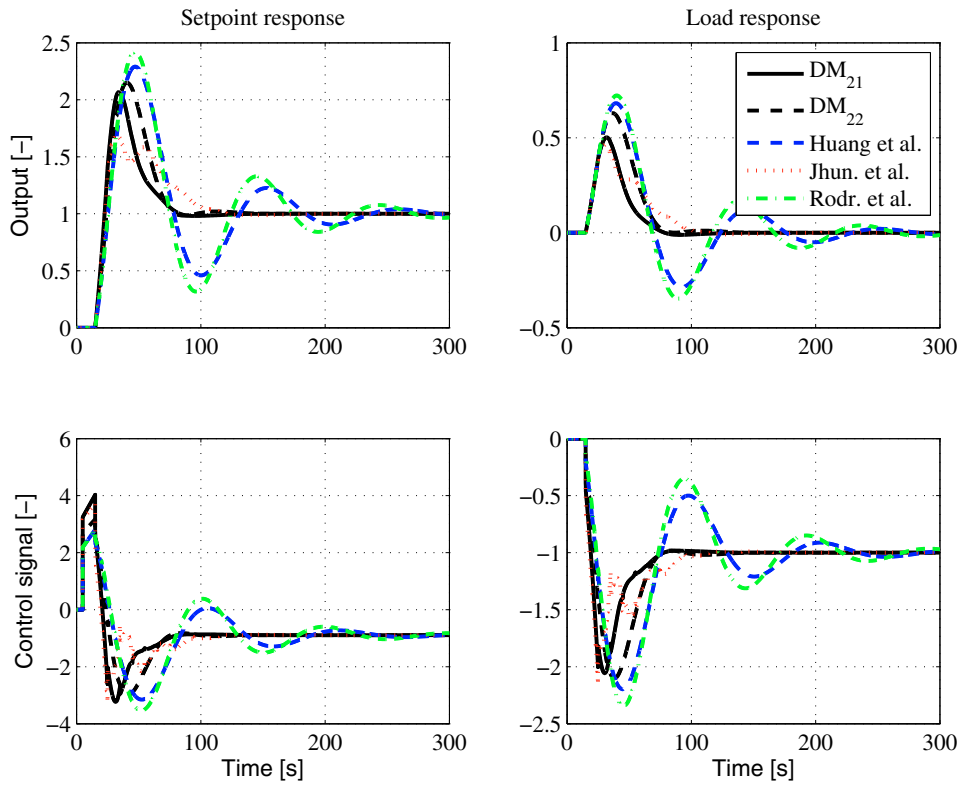
Fig. 3. Pareto set and front approximations for Cholette example. Selected solution after the MCDM process appears within a circle.

4. COMPARATIVE STUDY AND VALIDATION

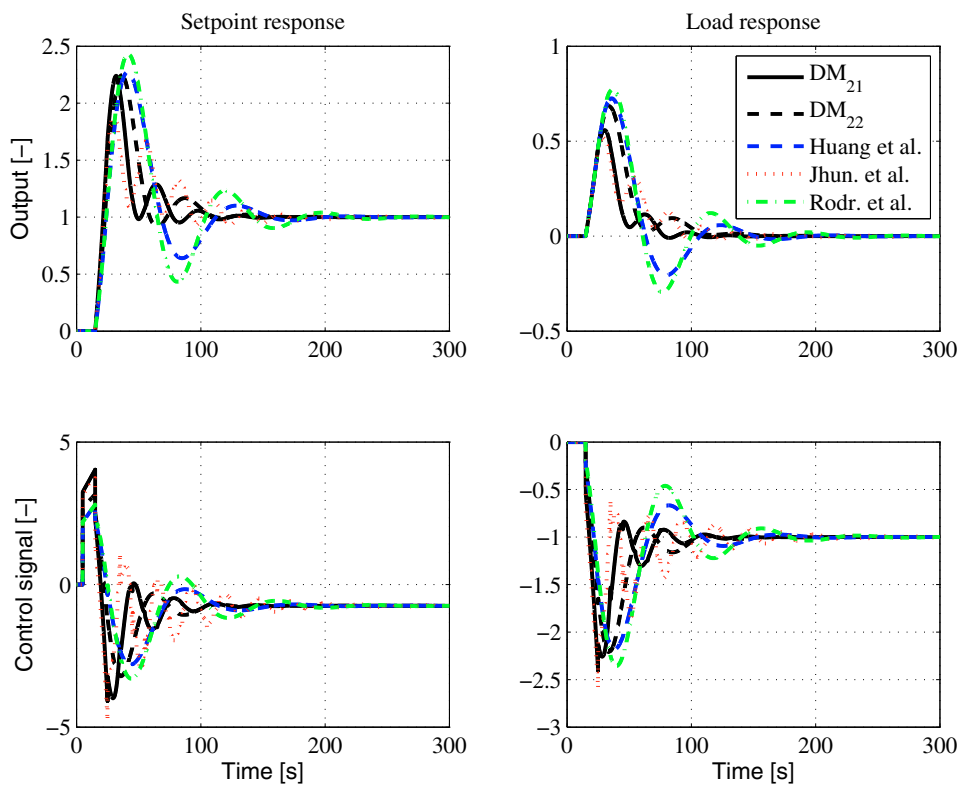
In order to validate the MOOD procedure exposed here, an unstable process will be used. With the aim of compensating the stochasticity added by the optimisation algorithm (an evolutionary algorithm), a total of 51 runs were carried out; afterwards, the Pareto front approximation with the median value of the Hypervolume ($HypV$) index⁴ is selected for further analysis. The Hypervolume measure is an usual choice to evaluate the performance of a given Pareto front approximation (Zitzler et al., 2003). This will be done in order to work with the *expected* performance of this approach. Simulations⁵ and Pareto front approximations were performed in a standard personal computer with Intel

⁴ that is, the volume enclosed by the Pareto front approximation and the reference point \mathbf{J}_{ref} . In this case, pertinency bounds are stated with such reference point.

⁵ Simulink©, with ode3 (Bogacki-Shampine) with fixed-step size of 10 ms



(a) Original Control loop



(b) Gain increase of 1.2

Fig. 4. Time response comparison for Cholette process.

Core i5-4210U, 1.7GHz, 4GB RAM. PID implemented for time response simulations uses the standard setpoint

weighting for the derivative action equals to zero, in order to avoid the *derivative kick* in step response.

An approximation for the Cholette reactor (Liou and Yu-Shu, 1991) is selected for evaluation; such process has been widely used in order to validate several control techniques as PI controllers (Ibarra-Junquera and Rosu, 2007) and Pareto-optimality techniques (Carrillo-Ahumada et al., 2011). For this work, an identified process around a stationary state with the feed concentration $c_{fs} = 3.2888[kmol \cdot m^3]$ and substrate concentration $c_s = 1.0439[kmol \cdot m^3]$ is used:

$$P_2(s) = \frac{1.121}{33.635s - 1} e^{-10s} \quad (12)$$

Pertinency constraints are defined as $J_{St}(\theta) \leq 150$, $J_{Gm-1}(\theta) \leq 0.5$ and $J_{Pm-1}(\theta) \leq 0.05$ while bound constraints as $k_p \in [0, 5]$, $T_I \in [1, 100]$, $T_D \in [0, 10]$. The Pareto front approximation with the *HypV* measure is depicted in Figure 3 using the Level Diagrams visualisation. For the decision making process, for this case, we are looking for controllers with $J_{St}(\theta) \leq 80[s]$ (depicted as dark solutions in Figure 3a, 3b). Using the color coding provided by the toolbox (black and gray in the same Figure), it is possible to identify the exchange of such solutions regarding gain and phase margin. Controller selected is identified within a circle (DM_{21}).

In order to validate the performance of this controller, it will be compared with controllers from Huang and Chen (1999) (H.etAl), Jhunjhunwala and Chidambaram (2001) (J.etAl) and Rodríguez-Mariano et al. (2015) (R.etAl), depicted in Table 1. Time performance is depicted in Figure 4 for two cases: (1) the original control loop and (2) a control loop with gain increased by a factor of 1.2; some additional time response indicators are shown in Figure 5. As it is possible to notice, the selected controller DM_{11} has a good performance on the nominal model, but a poor performance when compared with other techniques when gain is increased. Therefore, the decision maker might ponder his/her decision on selecting this controller from the approximated set; for example, it is more preferable to have a controller with the same level of performance in the nominal model, but with better robustness. This selection is depicted in Figure 3 with a square (DM_{22}), where a controller with almost the same performance in $J_{St}(\theta)$ than DM_{21} but a better $J_{Gm-1}(\theta)$ and worse $J_{Pm-1}(\theta)$. As it is possible to notice, this solution has a slightly different performance in the nominal model (Figure 5a), but a more acceptable performance when gain is increased (Figure 5b).

Table 1. PID Parameters for Cholette example.

Tuning	k_p	T_i	T_d
DM_{21}	3.24	41.66	4.11
DM_{22}	2.68	55.72	2.34
H.etAl	2.19	35.35	2.89
J.etAl	3.15	53.85	5.86
R.etAl	2.19	35.35	2.00

5. CONCLUSIONS AND FUTURE WORK

In this work, a multi-objective optimisation design (MOOD) procedure for unstable process, using PID controllers, was provided. This procedure is a promising tool for this kind

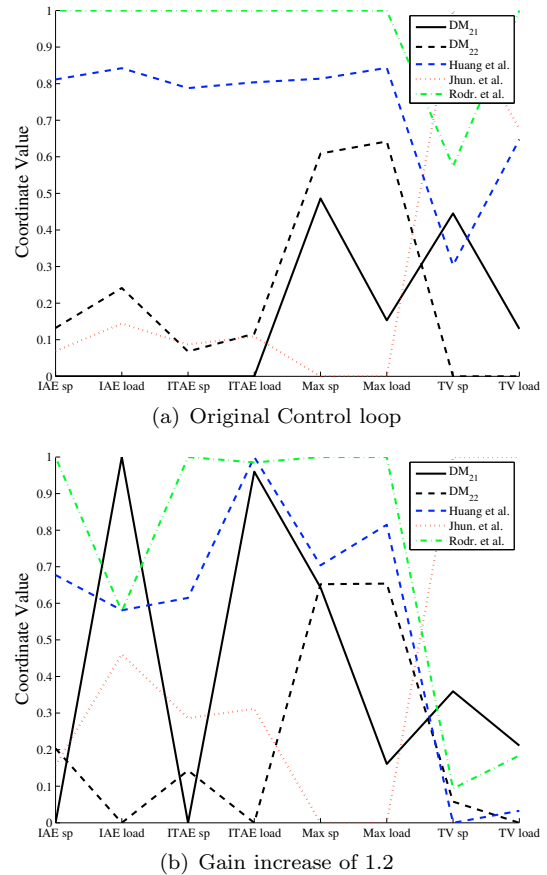


Fig. 5. Normalized controllers performance: integral of the absolute value of the error (IAE), integral of the time weighted absolute value of the error (ITAE), maximum weighted deviation (Max) and total variation (TV) of control action are depicted for setpoint (sp) and load response.

of process, sometimes difficult to control. The main advantage rely on the capabilities of analysing trade-off between conflictive design objectives, and therefore, this enables the designer to select a controller, fulfilling her/his wishes and preferences, regarding the typical trade-off between performance and robustness. Future work will focus on dealing with many-objectives optimisation instance and defining properly feasibility regions for the search process.

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